2016 Virginia Tech High School Contest
Solution Outlines

The Judges

Dec 10, 2016

A – Number Fun – First solved at 0:03

Problem

Given three small positive numbers a, b, c, find out whether combining two with an operation yields the third.

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Problem

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Solution

- Simply check whether $a + b = c$, $a - b = c$, $b - a = c$, $ab = c$, $a/b = c$, or $b/a = c$.
- Care must be taken when using integers in Java/C++.

B – Linden Mayor System – First solved at 0:09

Problem

Given a starting string, a set of rules, and a number m, output the resulting string after applying the rules m times.

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Solution

- Build the next string by iterating the current string one character at a time.
- For each character c in the current string:
  - If a rule $c \to y$ exists, append y to the new string.
  - Otherwise, just append c to the new string.
- Repeat this m times and print the final string.

F – Cookie Cutters – First solved at 0:13

Problem

Given a polygon's points, scale it to a desired area A and translate the polygon so that it touches the positive x- and y- axes.

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Solution

- Compute polygon area A_0.
- Compute scaling factor $k = \sqrt{\frac{A_0}{A}}$
- Multiple all coordinates with k.
- Subtract $\min_i x_i$ from all x
- Ditto for y.

D– Cracking RSA – First solved at 0:16

Problem

We are given two numbers n and e where n is the product of distinct primes p and q. Factor n to find p and q, and then find d, the solution to $de \equiv 1 \bmod (p-1)(q-1)$

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Problem

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Solution

- Since n is guaranteed to have 2 prime factors less than 1000:
  - Loop through all values $p = 2, 3, 4, \ldots, 999$.
  - Stop when n is divisible by p and set $q = \frac{n}{p}$.
- Calculate the value $\varphi(n) = (p-1)(q-1)$.
- Because $d < \varphi(n) < (1000-1)(1000-1) < 10^6$ a complete search works for d.
  - Loop through all values $d = 1, 2, 3, \ldots, \varphi(n)$.
  - Stop and output d when $de \equiv 1 \bmod \varphi(n)$. (Equivalently, stop when $de - 1 = k\varphi(n)$ for some integer k.)

G – Railroad – First solved at 0:19

Problem

Given a number of X-shaped level junctions and a number of Y-shaped switches, can you form a railroad without any dead-ends?

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Given a number of X-shaped level junctions and a number of Y-shaped switches, can you form a railroad without any dead-ends?

Solution

- If a railroad can be formed, it must form an undirected graph. The handshaking lemma states that

$$\sum_{v \in V} \deg(v) = 2|E|$$

  for vertices V. The sum of the degrees of all vertices must be even!
- Output possible if $4X + 3Y \equiv 0 \bmod 2$
- Equivalent to Y being even.

C – Turtle Master – First solved at 0:28

Problem

Move a robot turtle based on the instructions contained in a program.

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Problem

Move a robot turtle based on the instructions contained in a program.

Solution

- Carefully simulate the program, keeping track of the position and orientation of the turtle.
- Common way to express state: (r, c, d_r, d_c) where r is the current row, c the current column, and d_r, d_c are the deltas in the direction the turtle is facing.
- Moves can then be implemented simply, e.g. using Python syntax:
  - `F` $r, c = r + d_r, c + d_c$
  - `L` $d_r, d_c = -d_c, d_r$
  - `R` $d_r, d_c = d_c, -d_r$
- Be careful to check conditions (for firing laser, being at board boundaries, etc.)

J – Rock Climbing – First solved at 1:03

Problem

Given a rock-climbing wall made of squares that have positive or negative difficulty $w_{i,j}$, find the minimum initial energy needed to climb the wall.

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Solution

- Let $E_{i,j}$ be the minimum energy needed to make it from square (i, j) to one of the squares on the cliff. Initialize $E_{0,*} = 0$ and $E_{1\ldots,*} = \infty$
- Then, try to improve the estimate for the minimum energy by considering your neighbors and the cost to reach them.

$$E'_{i,j} = \max(0, \min(E_{i,j}, E_{i\pm1,j\pm1} + w_{i,j}))$$

- Iterate until $E = E'$. Answer is $\min E_{c,*}$.
- Variant of Bellman-Ford's algorithm.

I – Victory Through Synergy – First solved at 1:11

Problem

Determine if a perfect team is possible for a fantasy soccer team given the formation and the players on the team.

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Solution

- Model the formation as a graph so that edges can be retrieved when doing synergy calculations (i.e. adjacency list/matrix).
- Since there are $10! \approx 3.6$ million arrangements of players into nodes, we can simply try them all. Using backtracking (exhaustive search), try each arrangement.
- For each arrangement, calculate the synergy score for each player by using the incident edges on the player's node and check if the team is a perfect team.

E – Cowboy Checkers – First solved at 1:53

Problem

Check if there is a double-mill position for White in a game of Cowboy Checkers.

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Problem

Check if there is a double-mill position for White in a game of Cowboy Checkers.

Solution

- There are only 64 possible double mill positions.
- Check them all.
- Key to quick solution is to model double mills in a simple way.
- For instance, encode all possible mills, then consider combinations.
- Exploit horizontal/vertical symmetry.

H – Robot Turtles – First solved at 2:07

Problem

Given a map with various obstacles, find the shortest path to the goal.

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Given a map with various obstacles, find the shortest path to the goal.

Solution (1)

- Can model this a BFS (breadth-first search)
- State is denoted by (r, c, d_r, d_c, I_k) where I_k are the locations of remaining undestroyed ice castles.
- Works here because $|I_k| \leq 10$.

H – Robot Turtles – First solved at 2:07

Solution (2)

- Can model this as Dijkstra's shortest path.
- State is denoted by (m, r, c, d_r, d_c, P) where m is the number of moves and P is the path.
- Key insight: there's never a reason to shoot the laser unless the turtle moves onto the next square.